

On Comparing Precision Orbit Solutions of Geodetic Satellites Given Several Ocean Tide and Geopotential Models

John G. Warner * and Annie Lum *

US Naval Research Laboratory, Washington DC, 20375, United States

The ability to successfully perform many aspects of a satellite mission is directly impacted by the ability to precisely determine and predict the satellite's orbit through high precision orbit determination. The orbit determination process relies on numerical procedures, satellite measurements, and force modeling to generate the orbit solution and prediction. As a foundation for detailed force modeling, gravity forces due to the distribution of the Earth's mass must be accurately modeled. In prior research, a number of geopotential models and ocean tide models have been developed for use in orbit determination. This paper examines current and historically recommended geopotential and ocean tide models using the Naval Research Laboratory's Orbit Covariance Estimation and ANalysis tool. Geodetic satellites with high precision satellite laser ranging measurements are used as test cases to evaluate the predictive capabilities of the geopotential and ocean tide models. Orbit fit and prediction consistency metrics are generated for multiple geopotential and ocean tide model combinations. Results show that use of the EGM2008 geopotential model and the GOT4.8 or FES2004 ocean tide models generally result in predictive orbit solutions that more closely follow the definitive orbit solution. However, these results vary for different satellite orbits and time past the initial fit span.

I. Introduction

The ability to perform a range of satellite missions is directly impacted by the ability to precisely determine and predict a satellite's orbit. Detailed force modeling, including gravity forces due to the distribution of the Earth's mass, must be included in the orbit determination process to accurately determine and predict the satellite orbit. A number of geopotential models and ocean tide models have been developed and may be used to model gravity forces to varying degrees of success. The Naval Research Laboratory (NRL) Orbit Covariance Estimation and ANalysis (OCEAN) tool is used to evaluate the suitability of these models to the application of orbit determination.

OCEAN is a highly configurable, database driven software tool that enables precision orbit determination for a range of satellite missions. OCEAN allows users to simulate data, propagate a spacecraft state, or solve for an orbit using a Kalman Filter-Smoother (KFS) or Weighted Least Squares Orbit Determination (WLS-OD) process. Early history of OCEAN is given in Reference 1, while references 2, 3 and 4 discuss further developments. More recently OCEAN has been used to calculate orbits to support operations for the NRL UPPERSTAGE and TACSAT-4 satellite missions.

Previous work evaluates the suitability for older geopotential and ocean tide models in precision orbit determination^{5, 6}. In this paper, the currently recommended geopotential and ocean tide models are evaluated along with historically recommended models to better understand expected performance.

II. Orbit Determination Methodology

The International Laser Ranging Service (ILRS) provides global satellite laser ranging data in support of geodetic research activities.⁷ The ILRS catalogs Satellite Laser Ranging (SLR) data to a number of geodetic

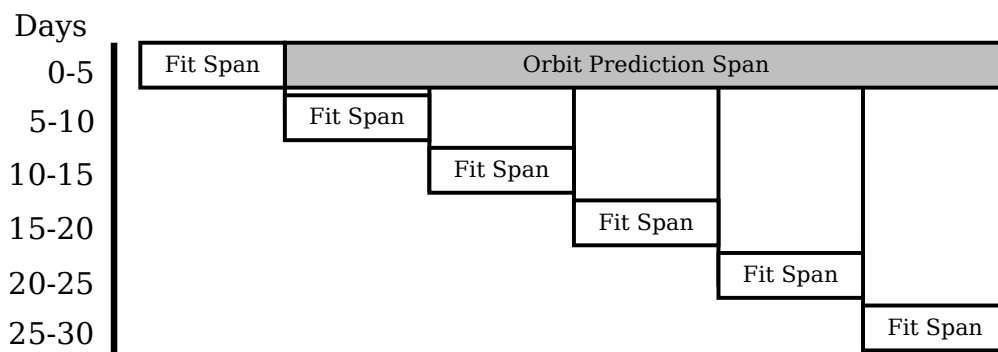
*Aerospace Engineer, Mission Development Branch, Washington, DC, AIAA Member

Report Documentation Page				Form Approved OMB No. 0704-0188		
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.						
1. REPORT DATE 01 AUG 2014		2. REPORT TYPE N/A		3. DATES COVERED -		
4. TITLE AND SUBTITLE On Comparing Precision Orbit Solutions of Geodetic Satellites Given Several Ocean Tide and Geopotential Models				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER		
				5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) John G. Warner Annie Lum				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory				8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)		
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release, distribution unlimited						
13. SUPPLEMENTARY NOTES This report was prepared for the AIAA 2014 SPACE Conference., The original document contains color images.						
14. ABSTRACT The ability to successfully perform many aspects of a satellite mission is directly impacted by the ability to precisely determine and predict the satellite's orbit through high precision orbit determination. The orbit determination process relies on numerical procedures, satellite measurements, and force modeling to generate the orbit solution and prediction. As a foundation for detailed force modeling, gravity forces due to the distribution of the Earth's mass must be accurately modeled. In prior research, a number of geopotential models and ocean tide models have been developed for use in orbit determination. This paper examines current and historically recommended geopotential and ocean tide models using the Naval Research Laboratory's Orbit Covariance Estimation and ANalysis tool. Geodetic satellites with high precision satellite laser ranging measurements are used as test cases to evaluate the solution accuracy and predictive capabilities of the geopotential and ocean tide models. Orbit fit and prediction comparison metrics are generated for multiple geopotential and ocean tide model combinations. Results show that use of the EGM2008 geopotential model combined with the GOT4.8 ocean tide model generally results in predictive orbit solutions that more closely follow the definitive orbit solution. However, these results vary for different satellite orbits and time past the initial fit span.						
15. SUBJECT TERMS Orbit determination, Astrodynamics, geopotential, geodesy						
16. SECURITY CLASSIFICATION OF:				17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 12	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified				

satellites. These satellites are typically designed to facilitate the study of Earth’s gravity and contain laser retro-reflectors to facilitate laser ranging data collection. High precision laser ranging measurements can then be used to perform precision orbit determination. ILRS guidelines call for a precision of the normal point laser range measurement to LAGEOS-1 of under one centimeter.⁸

OCEAN is used with SLR data to calculate precision orbits for several geodetic satellites including LAGEOS-1, LAGEOS-2, STARLETTE and STELLA. By comparing predicted orbits to fitted orbits for each satellite using a combination of geopotential and ocean tide models, the predictive consistency of the underlying models may be evaluated. Orbits are determined with the OCEAN Weighted Least Squares Orbit Determination (WLS-OD) methodology using successive five day increments of SLR data. The orbit solution from the first five day data arc is propagated forward in time to thirty days. The WLS-OD process is repeated for successive five day data arcs. These orbit solutions are then compared to the predicted orbit from the first data arc solution. Thirty days was chosen as a comparison time span to demonstrate the longer term variation in predictive consistency. The degree to which the predicted and fitted solutions agree will be used as a metric to evaluate the use of the geopotential and ocean tide models in precision orbit determination. Consistency (agreement) between predicted and fitted orbits using a particular geopotential and ocean tide model combination indicates that the force models are capturing the dynamics of the real system. The prediction/fit comparison methodology is depicted in Figure 1.

Figure 1: Depiction of Orbit Solution Comparison Methodology



OCEAN is further used to fit an orbit solution to thirty days of data. The RMS of the error residual is used as a metric to compare the relative effectiveness of the geopotential and ocean tide models to capture the long term variation in the satellite’s orbit. While data editing methodologies and raw data quality often have a large impact on the RMS of the error residuals, it is nonetheless used as a metric to compare orbit solution fit quality across the various models.

The OCEAN WLS-OD capability employs extensive spacecraft, measurement, and force modeling to estimate the desired spacecraft state and parameters. Force models include solid Earth tides; pole tides; lunar and solar third body gravitational effects; indirect lunar oblateness; general relativistic effects; atmospheric variability; drag; and solar radiation pressure. OCEAN WLS-OD accounts for various time systems, including TAI and UTC time. The WLS-OD functionality also uses Earth Orientation Parameter (EOP) models from the International Earth Rotation and Reference Systems Service (IERS), which account for precession and nutation, Earth rotation, and polar motion. For this application, a ninth order multi-step predictor-corrector algorithm is used to perform the integration of the state variables and state transition matrix. OCEAN follows the standards described in the IERS 2010 Conventions.⁹ OCEAN also performs iterative data editing to prevent low quality data from biasing the orbit solution.

A. LAGEOS-1

The LAGEOS-1 satellite was launched in 1976 by the National Aeronautics and Space Administration (NASA) as part of the Earth and Ocean Dynamics application program. It was designed to provide a long-lasting laser target in a well defined orbit. The LAGEOS-1 satellite enabled researchers to study a range of geophysical phenomena with improved accuracy, including the Earth’s geopotential^{10, 11}. The satellite’s low ballistic coefficient combined with its spherical shape minimize the orbital uncertainty due to drag and solar radiation forces.

The nominal orbital elements are given in Table 1.

Table 1: Nominal Orbital Elements for LAGEOS-1

Element	Nominal Value
Semi-major Axis	12,240 km
Eccentricity	0.0045
Inclination	109.84°

B. LAGEOS-2

The LAGEOS-2 satellite was launched in 1992 by NASA and the Agenzia Spaziale Italiana. The LAGEOS-2 satellite is almost identical in design to the LAGEOS-1 satellite; however, its orbit was selected to provide increased coverage over seismically active areas such as the Mediterranean and California.¹²

The nominal orbital elements for LAGEOS-2 are given in Table 2.

Table 2: Nominal Orbital Elements for LAGEOS-2

Element	Nominal Value
Semi-major Axis	11,998 km
Eccentricity	0.0135
Inclination	52.64°

C. STARLETTE

The STARLETTE satellite was launched in 1975 by Centre Nationale d'Etudes Saptiales (CNES). The spacecraft was designed to improve the geopotential model and to study solid Earth tides, ocean tides, and polar motion.¹³ STARLETTE was also the first spacecraft to be entirely covered by laser corner reflectors which allow passive SLR observation capabilities.¹⁴ The STARLETTE orbit is highly sensitive to temporal and zonal variations in the gravity field.

The nominal orbital elements for STARLETTE are given in Table 3.

Table 3: Nominal Orbital Elements for STARLETTE

Element	Nominal Value
Semi-major Axis	7,190 km
Eccentricity	0.0206
Inclination	49.83°

D. STELLA

The STELLA satellite was launched in 1993 by CNES, and is virtually identical to the STARLETTE satellite. However, it provides additional coverage over the polar regions due to its inclination. As with the LAGEOS satellites, both STELLA and STARLETTE are spherically-shaped spacecraft with low ballistic coefficients to minimize orbital uncertainty caused by drag and solar radiation pressure forces.¹⁵

The nominal orbital elements for the STELLA satellite are given in Table 4.

III. Geopotential Models

The International Center for Global Earth Models (ICGEM), which is a part of the International Association of Geodesy (IAG), catalogs Earth geopotential models. The catalog contains over 130 models which

Table 4: Nominal Orbital Elements for STELLA

Element	Nominal Value
Semi-major Axis	7,178 km
Eccentricity	0.0206
Inclination	98.6°

have been developed from as early as 1966 to the present day. Several geopotential models have been used historically for precision orbit determination. The International Earth Rotation and Reference Systems Service (IERS) serves the astronomical and geodetic communities by providing relevant data and maintaining standards and best practices. The IERS releases technical notes on a regular basis that define best practices for Earth rotation computation, modeling and analysis. Historically, the IERS has recommended the use of the Earth Gravitational Model 1996 (EGM96) geopotential¹⁶ and currently recommends the use of the Earth Gravitational Model 2008 (EGM2008) geopotential model.⁹ The EGM2008 model was developed using data from the Gravity Recovery and Climate Experiment (GRACE) as well as ocean altimetry and surface gravity data.¹⁷

Additional geopotential models have been developed using data obtained from the GRACE mission. Principal investigators for GRACE at the University of Texas at Austin Center for Space Research (CSR) have developed the GRACE Intermediate Field Model, GIF48,¹⁸ which was developed using GRACE data as well as ocean altimetry and surface gravity data.¹⁸ Researchers at CSR have also developed the Grace Gravity Model (GGM) geopotential series. The first GGM series model (GGM01) was released in 2003.¹⁹ Updates to the GGM01 model were made in GGM02²⁰ and GGM03,²¹ released in 2004 and 2008, respectively. The latest in the GGM series available from ICGEM is GGM05S. This model was developed using GRACE data only.²²

Further, researchers from the German Research Center for Geosciences and the Groupe de Recherche de Geodesie Spatiale have developed a series of gravity models under the European Improved Gravity model of the Earth using New methods (EIGEN) name. This model series makes use of CHAMP, GRACE and LAGEOS satellite data, as well as ocean altimetry and surface gravity data. The most recent is a static model named EIGEN6C3S.²³

Five geopotential models are used in this evaluation: EGM96, EGM2008, GIF48, GGM05S and EIGEN6C3S.

IV. Ocean Tide Models

The geopotential models capture gravity forces resulting from the distribution of Earth’s solid mass. Satellite orbits are additionally affected by gravity forces resulting from the distribution of Earth’s ocean mass. The IERS has historically recommended the use of the Center for Space Research CSR3.0 ocean tide model^{16,24} and currently recommends the Finite Element Solution 2004 (FES2004) model.⁹ FES2004 was produced by Legos and CLS Space Oceanography Division and distributed by Aviso, with support from CNES.^{25,26} Both of these models have been incorporated into OCEAN to maintain adherence to IERS recommendations for ocean tide modeling.

Other ocean tide models have also been developed from various satellite data. The CSR4.0 model represents an incremental improvement to the CSR3.0 model.²⁷ The Global Ocean Tide Model 4.8 (GOT4.8) is the latest in a series of models that use TOPEX/Poseidon and other satellite altimetry data,²⁸ and improves over GOT4.7 due to better processing of the dry component of the troposphere. Both of these models have been implemented within OCEAN.

Four ocean tide models will be used in this evaluation: CSR3.0, CSR4.0, FES2004, GOT4.8.

V. Testing Results

As can be anticipated, when examining the combined performance for five geopotential models and four ocean tide models in determining orbits for four satellites, a large number of results are generated. Presenting these data in a tractable manner presents a challenge. Since the EGM96 and EGM2008 geopotential models are recommended by the IERS technical conventions, the results presented are focused on these models.

However, select results for other geopotential models will be presented as necessary to draw additional conclusions.

Separate results are presented for each satellite considered. For each satellite, RMS residual errors are given for each fit span as a measure of orbit solution quality. In addition, RSS errors between the orbit prediction and the current interval orbit solution are given to show the predictive consistency of the specific geopotential and ocean tide model combination. Finally, plots of average daily RSS position difference between the predicted and fit orbits are given for select cases as a metric to evaluate the consistency of the underlying geopotential and ocean models for precision orbit determination.

A. LAGEOS-1 Results

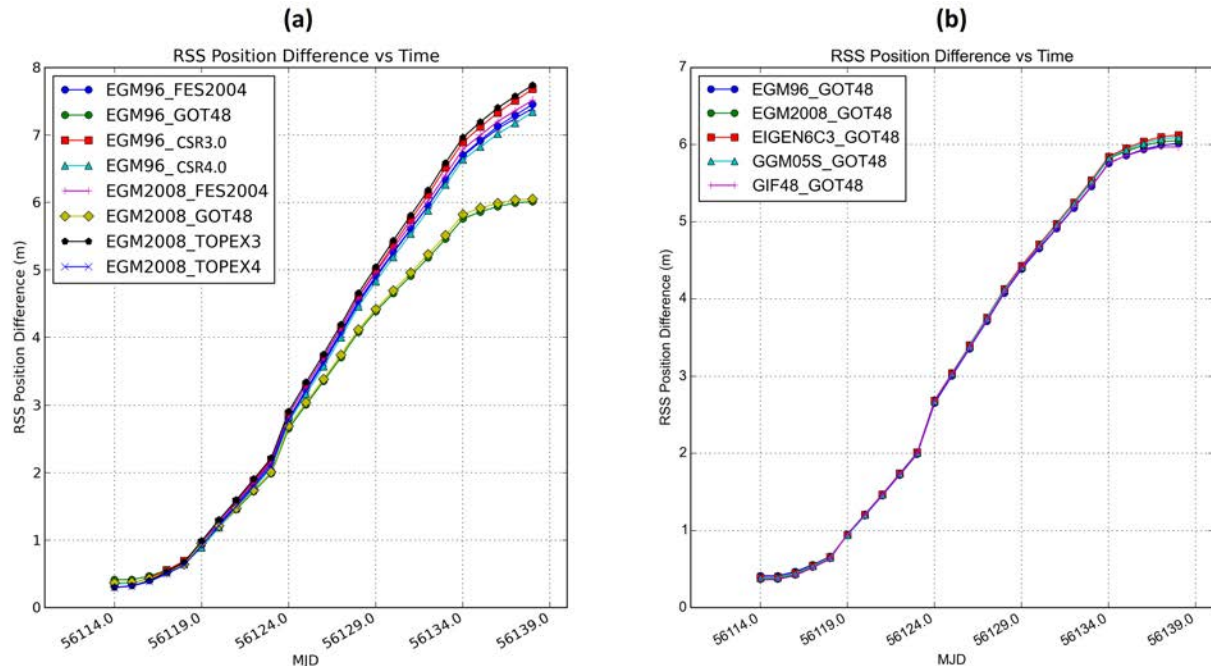
Table 5 shows the RMS of the residuals for a 30-day orbit solution given the specified geopotential model and ocean tide model. This may be used as a general comparison to determine relative performance between the competing models.

Table 5: LAGEOS-1 RMS Error Residuals for 30-day Orbit Fit in Meters

Geopotential Model	CSR3.0	CSR4.0	FES2004	GOT4.8
EGM96	0.4835	0.4840	0.4884	0.5457
EGM2008	0.5038	0.5048	0.5100	0.5629
GIF48	0.5132	0.5143	0.5198	0.5720
GGMS05S	0.4902	0.4910	0.4958	0.5512
EIGEN6C3S	0.4877	0.4884	0.4932	0.5494

As can be seen in Table 5, the lowest residuals for the 30-day orbit solution are found when the EGM96 geopotential and the CSR3.0 ocean tide model are used. However, these models do not produce most self-consistent orbit prediction when subsequent five day fits are examined.

Figure 2: LAGEOS-1 Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit (Left) and LAGEOS-1 Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit For GOT4.8 Ocean Tide Model and Various Geopotential Models (Right)



Next, Figure 2 (a) shows the average daily RSS difference between the orbit predicted in the first five

day fit compared to the orbit solution during the subsequent five day fits. It can be seen in Figure 2 (a) that both the EGM96 and EGM2008 geopotential models have substantially lower error when used with the GOT4.8 ocean tide model.

To demonstrate the impact of the geopotential model, results are presented for all the considered geopotential models used with the GOT4.8 ocean tide model in Figure 2 (b). It can be seen in Figure 2 (b) that there is little difference between the orbit prediction quality using any of the considered geopotential models for a fixed ocean tide model.

B. LAGEOS-2 Results

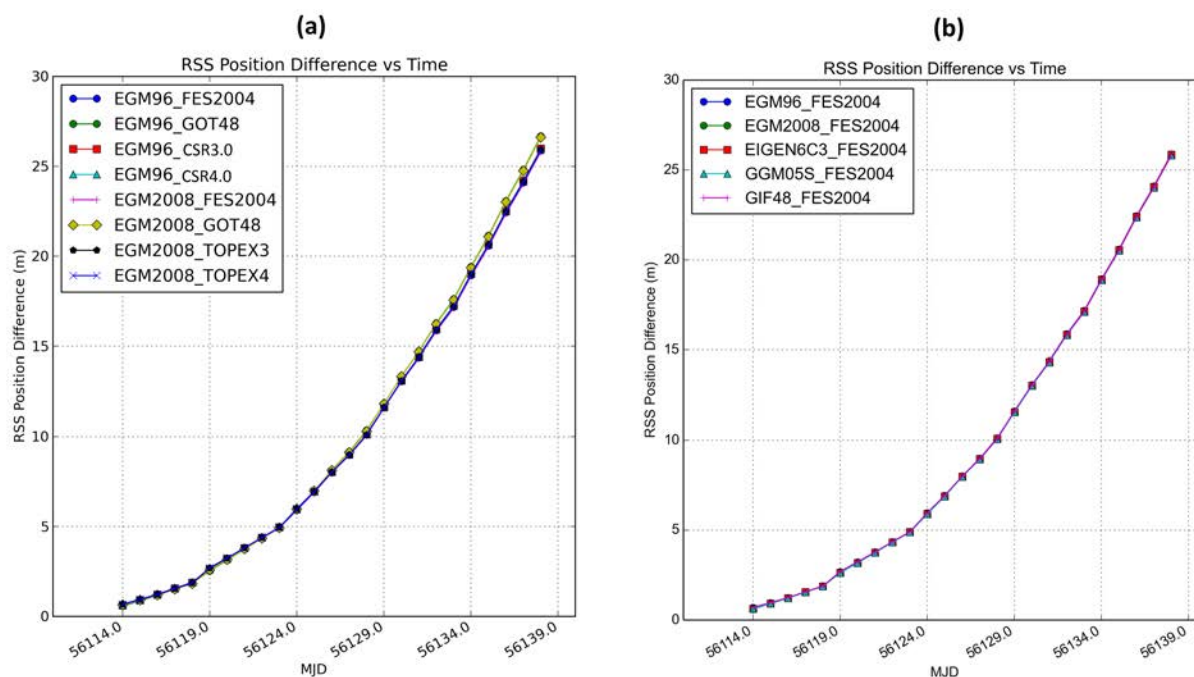
Table 6 shows the RMS of the residuals for a 30-day orbit solution given the specified geopotential model and ocean tide model.

Table 6: LAGEOS-2 RMS Error Residuals for 30-day Orbit Fit

Geopotential Model	CSR3.0	CSR4.0	FES2004	GOT4.8
EGM96	0.4937	0.4935	0.4939	0.4689
EGM2008	0.5855	0.5856	0.5859	0.5587
GIF48	0.6153	0.6156	0.6157	0.5878
GGMS05S	0.5306	0.5304	0.5309	0.5048
EIGEN6C3S	0.5087	0.5087	0.5089	0.4825

As can be seen from Table 6, the 30-day orbit solution residuals are the lowest for the EGM96 geopotential when used with the GOT4.8 ocean tide model. However, these models do not produce the lowest difference between orbit prediction and subsequent five day fits.

Figure 3: LAGEOS-2 Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit (Left) and LAGEOS-2 Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit for FES2004 Ocean Tide Model and Various Geopotential Models (Right)



Next, Figure 3 (a) shows the average daily RSS position difference between the orbit predicted from the first five day fit compared to the orbit solution during the subsequent five day fits. As can be seen in Figure 3 (a), there is no appreciable difference between the orbit prediction quality of the considered geopotential or

ocean tide models. The GOT4.8 ocean tide model provides the lowest prediction-fit difference for intervals 2 and 3 (Days 5-15 in Figure 1). However, the FES2004 ocean tide model provides slightly better prediction-fit consistency for prediction intervals 4, 5 and 6. Figure 3 (b) shows that there is little difference between the orbit prediction quality for the different geopotential models when using the FES2004 ocean tide model.

C. STARLETTE Results

Table 7 shows the RMS of the residuals for a 30-day orbit solution given the specified geopotential model and ocean tide model.

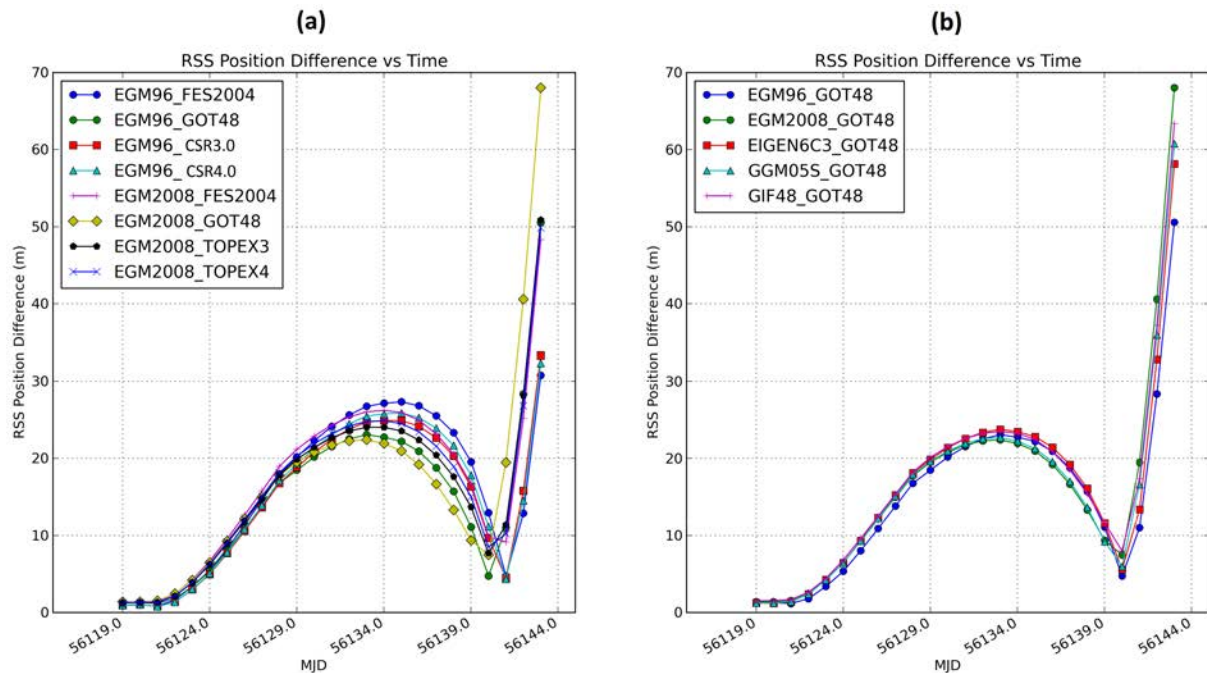
Table 7: STARLETTE RMS Error Residuals for 30-day Orbit Fit

Geopotential Model	CSR3.0	CSR4.0	FES2004	GOT4.8
EGM96	3.8037	3.7848	3.8017	3.8214
EGM2008	4.2087	4.1852	4.2059	4.2133
GIF48	4.2841	4.2606	4.2818	4.2850
GGMS05S	4.0977	4.0746	4.0945	4.1076
EIGEN6C3S	3.9332	3.9119	3.9308	3.9472

As can be seen in Table 3, the minimum RMS residual error for the 30-day orbit solution occurs when the EGM96 geopotential and the CSR4.0 ocean tide model are used. The RMS errors are higher in than the LAGEOS-1 and LAGEOS-2 test cases. The orbit of STARLETTE was designed to be sensitive to tidal variations as well as temporal variations in Earth's gravity. As a result, it is more difficult to capture STARLETTE's dynamics over a larger time span. However, the orbital dynamics are more easily captured for shorter time spans; and so, the RMS error for the five day fits are approximately 0.5 meters. Therefore, the predicted orbit versus fit orbit comparison is still an applicable metric.

Next, Figure 4 (a) shows the average daily RSS between the orbit predicted from the first five day fit compared to the orbit solution during the subsequent five day fits.

Figure 4: STARLETTE Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit (Left) and STARLETTE Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit for GOT4.8 Ocean Tide Model and Various Geopotential Models (Right)



As can be seen in Figure 4 (a), there is some variation in the RSS orbit difference between predicted and fitted orbits for the considered geopotential and ocean tide models. The combination of the EGM2008 geopotential with the GOT4.8 tide model provides the lowest prediction-fit orbit difference through most of the prediction intervals. However, at the final interval the best predictive performance is given by the combination of the EGM96 geopotential with the FES2004 ocean tide model.

Figure 4 (b) shows the RSS orbit difference between the predicted orbit and proceeding five day orbit fits for the GOT4.8 ocean tide model and all the considered geopotential models. As can be seen, the differences between the various geopotential models are relatively small. However, either EGM96 or EGM2008 produce orbits with the lowest RMS difference depending on the fit span.

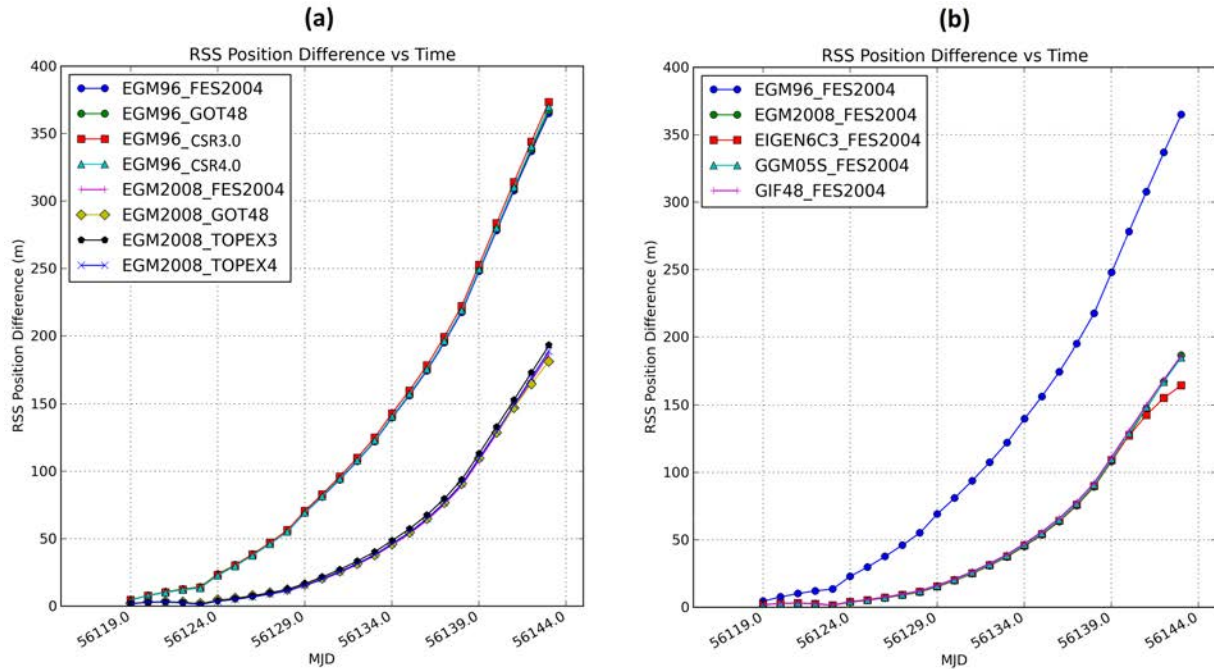
D. STELLA Results

Table 8 shows the RMS of the residuals for a 30-day orbit solution given the specified geopotential model and ocean tide model.

Table 8: STELLA RMS Error Residuals for 30-day Orbit Fit

Geopotential Model	CSR3.0	CSR4.0	FES2004	GOT4.8
EGM96	5.5905	5.6136	5.6479	5.7992
EGM2008	6.1335	6.1487	6.1670	6.3505
GIF48	6.1125	6.1284	6.1470	6.3286
GGMS05S	6.1408	6.1555	6.1735	6.3586
EIGEN6C3S	6.1295	6.1458	6.1685	6.3509

Figure 5: STELLA Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit (Left) and STELLA Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit For FES2004 Ocean Tide Model and Various Geopotentials (Right)



As can be seen in Table 8, the lowest RMS error is given by the EGM96 geopotential and the CSR3.0 ocean tide model. In addition, the 30-day orbit solution residuals are much higher than for either LAGEOS-1 or LAGEOS-2. This is largely due to the fact that the STELLA orbit was chosen to be particularly sensitive to the solid Earth and ocean tides as well as the temporal variations of the geopotential. Thus, the longer

fit orbit solution is not able to sufficiently model the sensitivity to these time varying forces, causing larger residuals. However, it is seen that with the shorter five day orbit fits, the RMS residuals are much lower at approximately 0.5 meters RMS or less. Since the dynamics of STELLA are more readily captured by the shorter data span, the comparison of successive five day orbit fits is still useful. As in previous cases, the set of models that minimizes the predict-fit orbit difference for the STELLA orbit is not the same as the models that minimize the 30-day residuals.

Figure 5 (a) shows the average daily RSS between the orbit predicted from the first five day fit compared to the orbit solution during the subsequent five day fits. As can be seen in Figure 5 (a), the EGM2008 geopotential model performs much better than the EGM96 geopotential model regardless of the ocean tide model used. There is only a small variation in orbit prediction consistency caused by the ocean tide model. However, over most prediction intervals, the combination of EGM2008 and FES2004 gives the lowest prediction-fit orbit difference.

Figure 5 (b) shows the average daily RSS error between the predicted orbit from the first five day orbit solution and the proceeding five day orbit solutions using the FES2004 ocean tide model and all of the considered geopotential models. As can be seen in Figure 5 (b), all geopotential models provide an orbit solution with similar quality prediction, except for the EGM96 geopotential model, which performs much worse than the other models. The EGM2008 geopotential model shows slightly better predictive consistency than GIG48, GGM05S, and EIGEN6C3 through most intervals. However, in the final interval, EIGEN6C3S provides better performance than EGM2008.

VI. Conclusions

There are a number of results between the four satellites, four ocean tide models and five geopotential models considered. From these results, there are some conclusions that may be drawn.

In the case of LAGEOS-1, it is seen that the use of the GOT4.8 ocean tide model produces orbit predictions 20 to 30 days ahead in time which are more consistent with SLR data than orbits produced using other tide models. The variation in predictive consistency due to the geopotential model is quite small when the GOT4.8 ocean tide model is used.

In the case of LAGEOS-2, there is very little variation in the predictive performance of the orbit solutions regardless of the models used.

In the case of STARLETTE, the combination of the EGM2008 geopotential and the GOT4.8 has the best predictive performance over the fourth and fifth intervals, where the orbit difference is the most pronounced.

In the case of STELLA, it can be seen that the EGM96 geopotential has the poorest predictive performance. There is little variation when any other geopotential model is used with any ocean tide model.

These results show that, for these test cases, the GOT4.8 and FES2004 ocean tide models along with the EGM2008 geopotential generally result in predictive orbits that more closely follow the definitive solutions based on additional SLR data. However, results vary depending on the orbit of the satellite and the time past the initial fit span.

Ultimately, these results provide a relative benchmark of the geopotential and ocean tide models for the predictive performance of precision orbit determination solutions. This is important for satellite mission planners who often require accurate predicted orbits. Future work includes examining other models, incorporating more geodetic satellites and examining the time variability of these results. Additionally, the comparison metrics in this analysis provide a level of confidence in a model's ability to capture real world dynamics by examining the consistency between a predicted orbit and a fit orbit using the same force models. Future research can expand on this to include a comparison of the fit orbit to an ILRS reference orbit as a measure of accuracy.

VII. Appendix

Orbit comparison results for all geopotential and ocean tide model combinations are given. Results are generated by comparing the predicted orbit solution to the fit orbit solution for the desired interval, as shown in Figure 1. Figure 6 shows the RMS of the radial, in-track, and cross-track position differences, along with the RSS position difference for LAGEOS-1 over each five day interval for the full set of geopotential and ocean tide model combinations examined in this paper. Figures 7, 8, and 9 provide the same information for LAGEOS-2, STARLETTE, and STELLA, respectively.

Figure 6: LAGEOS-1 Difference Between Predicted and Fit Orbits - Results Summary

	Geopotential Model	FES2004 Position Difference				GOT48 Position Difference				TOPEX3 Position Difference				TOPEX4 Position Difference			
		Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS
Interval 2	EGM96	0.052	0.361	0.348	0.504	0.047	0.319	0.426	0.535	0.055	0.377	0.344	0.513	0.055	0.349	0.346	0.494
	EGM2008	0.056	0.366	0.292	0.471	0.050	0.323	0.376	0.499	0.059	0.382	0.283	0.479	0.058	0.354	0.287	0.459
	GIF48	0.055	0.359	0.283	0.460	0.049	0.317	0.368	0.488	0.057	0.375	0.273	0.467	0.057	0.346	0.277	0.447
	GGM5055	0.053	0.363	0.315	0.484	0.047	0.321	0.397	0.513	0.056	0.379	0.309	0.492	0.055	0.351	0.312	0.472
	EIGEN6C3	0.052	0.364	0.322	0.489	0.047	0.322	0.404	0.519	0.055	0.380	0.317	0.498	0.055	0.352	0.320	0.479
Interval 3	EGM96	0.049	1.556	0.332	1.592	0.029	1.409	0.540	1.509	0.053	1.604	0.310	1.634	0.052	1.505	0.313	1.538
	EGM2008	0.056	1.572	0.395	1.622	0.035	1.424	0.544	1.525	0.059	1.620	0.369	1.663	0.059	1.520	0.368	1.566
	GIF48	0.051	1.555	0.443	1.618	0.034	1.407	0.569	1.518	0.055	1.603	0.418	1.657	0.054	1.503	0.416	1.561
	GGM5055	0.051	1.575	0.341	1.612	0.031	1.426	0.526	1.520	0.055	1.622	0.315	1.654	0.054	1.523	0.316	1.556
	EIGEN6C3	0.050	1.579	0.334	1.615	0.030	1.431	0.526	1.525	0.053	1.627	0.308	1.657	0.053	1.527	0.310	1.559
Interval 4	EGM96	0.084	3.665	0.577	3.711	0.042	3.281	0.874	3.396	0.084	3.729	0.540	3.769	0.085	3.596	0.539	3.637
	EGM2008	0.090	3.687	0.757	3.765	0.050	3.300	0.937	3.431	0.091	3.749	0.722	3.819	0.091	3.612	0.718	3.683
	GIF48	0.081	3.648	0.838	3.744	0.041	3.263	0.986	3.409	0.082	3.711	0.805	3.798	0.083	3.575	0.801	3.664
	GGM5055	0.086	3.699	0.647	3.756	0.044	3.312	0.887	3.429	0.086	3.761	0.610	3.811	0.087	3.628	0.608	3.680
	EIGEN6C3	0.082	3.709	0.625	3.762	0.041	3.323	0.879	3.437	0.082	3.771	0.588	3.817	0.083	3.635	0.586	3.682
Interval 5	EGM96	0.097	5.557	0.899	5.630	0.040	4.761	1.266	4.926	0.097	5.701	0.851	5.765	0.099	5.489	0.851	5.556
	EGM2008	0.106	5.559	1.231	5.694	0.052	4.760	1.435	4.972	0.107	5.706	1.192	5.830	0.109	5.492	1.187	5.619
	GIF48	0.094	5.493	1.355	5.658	0.042	4.699	1.519	4.938	0.096	5.640	1.319	5.793	0.098	5.426	1.313	5.584
	GGM5055	0.100	5.594	1.046	5.691	0.044	4.794	1.326	4.974	0.101	5.750	1.002	5.838	0.103	5.528	1.000	5.619
	EIGEN6C3	0.095	5.613	1.005	5.703	0.040	4.817	1.305	4.990	0.096	5.760	0.960	5.840	0.098	5.548	0.957	5.631
Interval 6	EGM96	0.072	7.027	1.014	7.100	0.018	5.730	1.449	5.910	0.060	7.242	0.963	7.306	0.067	6.932	0.954	6.998
	EGM2008	0.075	7.025	1.445	7.172	0.010	5.727	1.651	5.960	0.066	7.238	1.408	7.374	0.071	6.924	1.391	7.063
	GIF48	0.059	6.928	1.599	7.110	0.009	5.629	1.753	5.896	0.051	7.141	1.566	7.310	0.056	6.826	1.548	7.000
	GGM5055	0.072	7.088	1.212	7.191	0.014	5.789	1.522	5.986	0.062	7.302	1.168	7.396	0.067	6.989	1.154	7.084
	EIGEN6C3	0.067	7.120	1.158	7.214	0.013	5.821	1.498	6.011	0.056	7.333	1.113	7.417	0.062	7.018	1.100	7.104

Figure 7: LAGEOS-2 Difference Between Predicted and Fit Orbits - Results Summary

	Geopotential Model	FES2004 Position Difference				GOT48 Position Difference				TOPEX3 Position Difference				TOPEX4 Position Difference			
		Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS
Interval 2	EGM96	0.080	1.231	0.503	1.333	0.079	1.197	0.460	1.285	0.082	1.212	0.504	1.315	0.083	1.227	0.503	1.329
	EGM2008	0.078	1.186	0.626	1.343	0.077	1.152	0.589	1.296	0.080	1.167	0.625	1.326	0.081	1.182	0.625	1.340
	GIF48	0.079	1.175	0.670	1.355	0.077	1.141	0.635	1.308	0.081	1.156	0.669	1.338	0.082	1.171	0.669	1.351
	GGM5055	0.077	1.200	0.552	1.323	0.076	1.166	0.513	1.276	0.079	1.181	0.552	1.306	0.080	1.196	0.552	1.320
	EIGEN6C3	0.077	1.214	0.532	1.328	0.075	1.181	0.492	1.281	0.079	1.195	0.532	1.311	0.080	1.210	0.532	1.324
Interval 3	EGM96	0.176	3.738	0.956	3.862	0.176	3.749	0.828	3.843	0.178	3.768	0.964	3.894	0.178	3.774	0.967	3.900
	EGM2008	0.180	3.658	1.274	3.878	0.180	3.670	1.163	3.854	0.182	3.688	1.281	3.909	0.182	3.694	1.284	3.915
	GIF48	0.182	3.635	1.377	3.891	0.181	3.646	1.271	3.866	0.184	3.665	1.384	3.922	0.184	3.670	1.388	3.928
	GGM5055	0.179	3.684	1.092	3.847	0.179	3.696	0.974	3.826	0.181	3.714	1.100	3.878	0.181	3.720	1.103	3.884
	EIGEN6C3	0.176	3.705	1.042	3.853	0.175	3.717	0.921	3.833	0.178	3.736	1.050	3.884	0.178	3.741	1.053	3.891
Interval 4	EGM96	0.318	7.958	1.637	8.131	0.326	8.138	1.433	8.269	0.320	7.986	1.654	8.162	0.319	7.979	1.656	8.155
	EGM2008	0.322	7.837	2.116	8.124	0.331	8.017	1.940	8.255	0.324	7.866	2.131	8.156	0.324	7.859	2.134	8.150
	GIF48	0.325	7.800	2.275	8.132	0.333	7.980	2.104	8.260	0.327	7.828	2.289	8.162	0.326	7.822	2.292	8.157
	GGM5055	0.321	7.880	1.844	8.099	0.330	8.060	1.655	8.235	0.323	7.908	1.860	8.130	0.322	7.901	1.862	8.124
	EIGEN6C3	0.316	7.914	1.770	8.115	0.324	8.094	1.577	8.252	0.318	7.942	1.787	8.147	0.317	7.935	1.789	8.141
Interval 5	EGM96	0.358	14.333	2.495	14.552	0.376	14.710	2.320	14.896	0.360	14.385	2.505	14.605	0.360	14.389	2.512	14.611
	EGM2008	0.362	14.160	3.206	14.523	0.381	14.544	3.059	14.867	0.365	14.217	3.213	14.580	0.365	14.221	3.221	14.586
	GIF48	0.363	14.116	3.434	14.533	0.382	14.505	3.293	14.879	0.366	14.169	3.441	14.585	0.366	14.180	3.449	14.598
	GGM5055	0.360	14.218	2.802	14.496	0.379	14.601	2.643	14.843	0.363	14.276	2.810	14.555	0.363	14.281	2.818	14.561
	EIGEN6C3	0.352	14.264	2.692	14.520	0.370	14.646	2.529	14.867	0.354	14.301	2.701	14.559	0.356	14.327	2.708	14.585
Interval 6	EGM96	0.472	22.255	3.279	22.500	0.509	22.923	3.155	23.145	0.473	22.356	3.291	22.602	0.472	22.340	3.300	22.588
	EGM2008	0.479	22.058	4.157	22.452	0.517	22.724	4.060	23.090	0.481	22.160	4.166	22.553	0.479	22.143	4.176	22.538
	GIF48	0.481	21.998	4.436	22.446	0.518	22.667	4.346	23.085	0.483	22.099	4.445	22.547	0.482	22.083	4.455	22.533
	GGM5055	0.477	22.128	3.659	22.434	0.514	22.796	3.551	23.077	0.478	22.230	3.670	22.536	0.476	22.213	3.679	22.521
	EIGEN6C3	0.467	22.190	3.525	22.473	0.503	22.856	3.413	23.115	0.468	22.291	3.536	22.575	0.467	22.273	3.546	22.559

Figure 8: STARLETTE Difference Between Predicted and Fit Orbits - Results Summary

	Geopotential Model	FES2004 Position Difference				GOT48 Position Difference				TOPEX3 Position Difference				TOPEX4 Position Difference			
		Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS
Interval 2	EGM96	0.047	1.657	0.854	1.865	0.085	1.601	1.230	2.021	0.042	1.463	0.813	1.675	0.042	1.483	0.796	1.684
	EGM2008	0.066	2.029	1.413	2.473	0.085	1.973	1.549	2.510	0.059	1.814	1.379	2.279	0.058	1.836	1.368	2.290
	GIF48	0.072	2.091	1.663	2.673	0.097	2.037	1.740	2.681	0.063	1.874	1.631	2.485	0.062	1.896	1.622	2.496
	GGMS05S	0.055	2.088	1.160	2.389	0.079	2.032	1.379	2.457	0.050	1.869	1.122	2.181	0.049	1.892	1.109	2.194
	EIGEN6C3	0.052	2.128	1.064	2.380	0.088	2.074	1.327	2.464	0.045	1.908	1.026	2.167	0.045	1.931	1.012	2.180
Interval 3	EGM96	0.169	12.196	2.075	12.372	0.383	11.543	2.053	11.731	0.171	11.341	2.039	11.525	0.166	11.580	2.010	11.754
	EGM2008	0.180	13.007	3.480	13.465	0.403	12.358	3.277	12.792	0.178	12.149	3.451	12.631	0.173	12.388	3.423	12.853
	GIF48	0.188	13.250	4.028	13.850	0.414	12.602	3.792	13.166	0.183	12.393	4.001	13.024	0.179	12.631	3.973	13.242
	GGMS05S	0.187	13.213	2.903	13.530	0.399	12.564	2.752	12.868	0.189	12.356	2.873	12.687	0.184	12.594	2.844	12.912
	EIGEN6C3	0.186	13.382	2.664	13.646	0.396	12.733	2.542	12.990	0.189	12.525	2.632	12.800	0.184	12.763	2.603	13.027
Interval 4	EGM96	0.349	23.670	3.265	23.896	0.647	21.013	2.974	21.232	0.351	21.983	3.166	22.212	0.327	22.588	3.145	22.808
	EGM2008	0.445	23.369	5.513	24.015	0.770	20.736	5.216	21.396	0.436	21.684	5.413	22.354	0.412	22.288	5.395	22.935
	GIF48	0.439	24.021	6.396	24.861	0.771	21.383	6.099	22.249	0.428	22.335	6.297	23.209	0.405	22.939	6.279	23.786
	GGMS05S	0.454	23.809	4.567	24.247	0.757	21.172	4.271	21.611	0.449	22.123	4.467	22.574	0.425	22.727	4.448	23.162
	EIGEN6C3	0.402	24.546	4.180	24.902	0.702	21.898	3.886	22.251	0.400	22.859	4.081	23.224	0.376	23.644	4.061	23.816
Interval 5	EGM96	0.470	25.808	3.771	26.087	0.638	19.969	3.479	20.280	0.483	23.194	3.620	23.479	0.456	24.306	3.607	24.577
	EGM2008	0.608	23.392	6.548	24.299	0.841	17.664	6.265	18.761	0.600	20.797	6.396	21.766	0.572	21.897	6.384	22.815
	GIF48	0.589	24.743	7.690	25.917	0.827	18.969	7.408	20.381	0.582	22.138	7.538	23.393	0.553	23.244	7.526	24.438
	GGMS05S	0.628	24.133	5.352	24.727	0.832	18.380	5.066	19.083	0.625	21.533	5.200	22.161	0.596	22.635	5.188	23.230
	EIGEN6C3	0.545	26.130	4.904	26.592	0.727	20.305	4.616	20.835	0.550	23.516	4.752	23.998	0.521	24.628	4.740	25.085
Interval 6	EGM96	0.306	18.408	2.981	18.650	0.526	27.047	2.656	27.182	0.326	18.692	2.814	18.906	0.314	18.584	2.802	18.796
	EGM2008	0.437	25.779	5.995	26.471	0.584	36.680	5.531	37.099	0.455	27.037	5.825	27.661	0.435	26.501	5.811	27.134
	GIF48	0.429	23.650	7.365	24.774	0.554	33.909	6.873	34.602	0.455	24.641	7.195	25.674	0.436	24.222	7.181	25.268
	GGMS05S	0.459	22.326	4.688	22.817	0.553	32.755	4.261	33.036	0.484	23.379	4.518	23.816	0.459	22.960	4.505	23.402
	EIGEN6C3	0.432	21.669	4.191	22.075	0.461	31.025	3.787	31.258	0.486	22.302	4.022	22.667	0.471	22.051	4.009	22.418

Figure 9: STELLA Difference Between Predicted and Fit Orbits - Results Summary

	Geopotential Model	FES2004 Position Difference				GOT48 Position Difference				TOPEX3 Position Difference				TOPEX4 Position Difference			
		Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS	Radial	In-Track	Cross-Track	RSS
Interval 2	EGM96	0.278	10.138	0.330	10.148	0.232	10.587	0.680	10.611	0.256	10.232	0.270	10.239	0.271	10.043	0.266	10.051
	EGM2008	0.107	2.354	0.483	2.405	0.159	2.762	0.782	2.875	0.160	2.445	0.407	2.484	0.143	2.302	0.409	2.342
	GIF48	0.082	2.523	0.393	2.554	0.134	2.939	0.768	3.041	0.135	2.615	0.327	2.638	0.119	2.464	0.323	2.488
	GGMS05S	0.124	2.470	0.535	2.530	0.177	2.885	0.795	2.997	0.177	2.564	0.456	2.610	0.159	2.416	0.461	2.464
	EIGEN6C3	0.056	2.361	0.553	2.426	0.105	2.768	0.798	2.883	0.107	2.449	0.474	2.496	0.092	2.305	0.479	2.356
Interval 3	EGM96	0.575	39.970	1.253	39.993	0.502	40.852	2.213	40.915	0.516	40.760	1.281	40.783	0.532	39.949	1.238	39.972
	EGM2008	0.315	7.625	0.973	7.693	0.395	8.508	1.769	8.699	0.476	8.452	0.916	8.515	0.451	7.643	0.899	7.709
	GIF48	0.272	8.388	0.917	8.442	0.351	9.271	1.820	9.454	0.435	9.212	0.880	9.265	0.409	8.402	0.853	8.455
	GGMS05S	0.349	8.060	1.014	8.131	0.429	8.946	1.754	9.127	0.510	8.889	0.950	8.954	0.484	8.079	0.938	8.148
	EIGEN6C3	0.210	7.799	1.035	7.870	0.287	8.679	1.743	8.857	0.374	8.621	0.968	8.683	0.350	7.811	0.958	7.877
Interval 4	EGM96	0.890	96.380	3.385	96.443	0.819	97.033	4.948	97.162	0.847	98.656	3.567	98.724	0.874	96.965	3.464	97.031
	EGM2008	0.565	26.730	1.325	26.769	0.637	27.364	2.943	27.530	0.748	29.014	1.369	29.056	0.707	27.334	1.299	27.374
	GIF48	0.500	28.268	1.509	28.313	0.570	28.906	3.189	29.086	0.684	30.555	1.602	30.604	0.644	28.873	1.516	28.920
	GGMS05S	0.613	27.541	1.282	27.578	0.685	28.179	2.857	28.332	0.794	29.827	1.302	29.866	0.753	28.147	1.242	28.185
	EIGEN6C3	0.409	27.096	1.257	27.128	0.477	27.729	2.798	27.874	0.597	29.377	1.260	29.410	0.558	27.699	1.207	27.731
Interval 5	EGM96	1.091	178.363	7.189	178.511	0.998	179.343	9.479	179.596	1.024	182.261	7.423	182.415	1.059	179.546	7.267	179.696
	EGM2008	0.679	67.109	2.988	67.179	0.764	68.091	5.460	68.314	0.926	70.989	3.172	71.066	0.875	68.288	3.023	68.360
	GIF48	0.603	69.634	3.448	69.722	0.685	70.619	5.908	70.869	0.852	73.520	3.646	73.615	0.802	70.818	3.494	70.909
	GGMS05S	0.744	68.396	2.833	68.459	0.829	69.381	5.305	69.589	0.989	72.279	3.010	72.349	0.938	69.578	2.863	69.643
	EIGEN6C3	0.494	67.762	2.722	67.819	0.573	68.744	5.194	68.942	0.746	71.643	2.895	71.705	0.697	68.943	2.749	69.001
Interval 6	EGM96	1.565	309.094	13.962	309.413	1.468	310.659	17.217	311.139	1.632	315.616	14.269	315.942	1.639	312.066	14.067	312.387
	EGM2008	1.581	149.643	7.503	149.840	1.674	148.004	10.760	148.404	1.937	155.247	7.748	155.453	1.886	151.513	7.538	151.712
	GIF48	1.496	151.481	8.052	151.703	1.489	154.302	11.489	154.737	1.792	159.745	8.421	159.977	1.798	153.485	8.096	153.709
	GGMS05S	1.614	149.873	7.225	150.056	1.635	151.788	10.641	152.170	1.977	155.219	7.458	155.410	1.945	150.618	7.208	150.803
	EIGEN6C3	1.590	140.783	6.701	140.951	1.418	150.470	10.471	150.840	1.761	154.182	7.295	154.365	1.688	151.611	7.138	151.789

References

- ¹Soyka, M. T., Middour, J. W., Binning, P. W., Pickard, H., and Fein, J., "The Naval Research Laboratory's orbit/covariance estimation and analysis software- OCEAN," *Astrodynamics 1997*, 1997, pp. 1567–1586.
- ²Soyka, M. T., Middour, J. W., and Fein, J., "Simultaneous orbit determination of large satellite constellations," *Spaceflight mechanics 1998*, 1998, pp. 1275–1293.
- ³Binning, P. W., Soyka, M. T., and Middour, J. W., "Orbit determination using space to ground Differential GPS in NRL's OCEAN package," *Astrodynamics 1999*, 2000, pp. 421–434.
- ⁴Seago, J. H., Davis, M. A., Smith, W., Fein, J., Brown, B., Middour, J., Soyka, M., and Lydick, E., "More results of naval space surveillance system calibration using satellite laser ranging," *Advances in the Astronautical Sciences*, Vol. 112, 2002, pp. 1177–1196.
- ⁵Kubo-Oka, T., Matsumoto, K., Otsubo, T., and Gotoh, T., "Effect of Ocean Tide Models on the Precise Orbit Determination of Geodetic Satellites," *AGU Fall Meeting Abstracts*, Vol. 1, 2005, Abstract G33A-0030.
- ⁶Casotto, S., "Nominal ocean tide models for TOPEX precise orbit determination," 1989.
- ⁷Pearlman, M., Degnan, J., and Bosworth, J., "The International Laser Ranging Service," *Advances in Space Research*, Vol. 30, No. 2, 2002, pp. 135 – 143.
- ⁸"ILRS System Performance Standards," http://ilrs.gsfc.nasa.gov/network/system_performance/index.html, 2014.
- ⁹Petit, G. and Luzum, B., "IERS conventions (2010)," Tech. rep., DTIC Document, 2010.
- ¹⁰Fitzmaurice, M., Minott, P., Abshire, J., and Rowe, H., "Prelaunch testing of the laser geodynamic satellite (LAGEOS)," *NASA STI/Recon Technical Report N*, Vol. 78, 1977, pp. 10200.
- ¹¹Siry, J., "The LAGEOS system," 1975.

- ¹²Minott, P. O., Zagwodzki, T. W., Varghese, T., and Seldon, M., "Prelaunch optical characterization of the Laser Geodynamic Satellite (LAGEOS 2)," *NASA STI/Recon Technical Report N*, Vol. 94, 1993, pp. 15193.
- ¹³Kanner, L. and Associates, "Translation of 'Le satellite de geodesie 'Starlette'," Groupe de Recherches de Geodesie Spatiale, Centre National d'Etudes Spatiales, Bretigny-sur-Orge, France, Report, 1974, 25 pp," National Aeronautics and Space Administration, Washington, D. C., July 1974.
- ¹⁴Kramer, H. J., *Observation of the Earth and Its Environment: Survey of Missions and Sensors*, Springer, 2002.
- ¹⁵Lefebvre, M., "New Satellite Missions for Solid Earth Studies-Status and Preparations," *CSTG Bulletin*, , No. 11, 1989, pp. 25–32.
- ¹⁶McCarthy, D. D. and Petit, G., "IERS conventions (2003)," Tech. rep., DTIC Document, 2004.
- ¹⁷Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K., "The development and evaluation of the Earth Gravitational Model 2008 (EGM2008)," *Journal of Geophysical Research: Solid Earth (1978–2012)*, Vol. 117, No. B4, 2012.
- ¹⁸Ries, J. C., Bettadpur, S. V., Poole, S., and Richter, T., "Mean background gravity fields for GRACE processing," Aug. 2011, GRACE Science Team Meeting, Austin, TX.
- ¹⁹Tapley, B. D., Bettadpur, S., Watkins, M., and Reigber, C., "The Gravity Recovery and Climate Experiment: Mission Overview and Early Results," *Geophysical Research Letters*, Vol. 31, No. 9, 2004.
- ²⁰Tapley, B., Ries, J., Bettadpur, S., Chambers, D., Cheng, M., Condi, F., Gunter, Z., Nagel, P., Pastor, R., Pekker, T., Poole, S., and Wang, F., "GGM02 - An improved Earth gravity field model from GRACE," *Journal of Geodesy*, Vol. 79, No. 8, pp. 467–478.
- ²¹Tapley, B., Ries, J., Bettadpur, S., Chambers, D., Cheng, M., Condi, F., and Poole, S., "The GGM03 Mean Earth Gravity Model from GRACE," *AGU Fall Meeting Abstracts*, 2007, Abstract G42A-03.
- ²²"Description of GRACE Gravity Model GGM05S," http://icgem.gfz-potsdam.de/ICGEM/documents/README_GGM05s.pdf, 2014.
- ²³Forste, C., Bruinsma, S., Flechtner, F., Abrykosov, O., Dahle, C., Marty, J. C., Lemoine, J. M., Biancale, R., Barthelmes, F., Neumayer, K. H., and König, R., "EIGEN-6C3 - The Latest Combined Global Gravity Field Model Including GOCE Data up to Degree and Order 1949 of GFZ Potsdam and GCGS Toulouse," *AGU Fall Meeting Abstracts*, 2013, Abstract G51A-0860.
- ²⁴Eanes, R. J. and Bettadpur, S., "The CSR 3.0 Ocean Tide Model: Diurnal and Semi-Diurnal Ocean Tides from Topex/Poseidon Altimetry," Center for Space Research Technical Memorandum, 1974.
- ²⁵"Description FES2004," <http://www.aviso.oceanobs.com>.
- ²⁶Lyard, F., Lefevre, F., Letellier, T., and Francis, O., "Modelling the global ocean tides: modern insights from FES2004," *Ocean Dynamics*, Vol. 56, No. 5-6, 2006, pp. 394–415.
- ²⁷Eanes, R. J., "The CSR 4.0 Ocean Tide Model," <ftp://www.csr.utexas.edu/pub/tide>, 2002.
- ²⁸Ray, R., "A Global Ocean Tide Model From TOPEX/POSEIDON Altimetry: GOT99.2," National Aeronautics and Space Administration Technical Memorandum, Sept. 1999.